

Quantum probes of timelike naked singularity with scalar hair

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ABSTRACT: We study the curvature singularity resolution via quantum fields on a fixed background based on Klein–Gordon and Dirac equations for a static spacetime with a scalar field producing a timelike naked singularity. We show that both Klein–Gordon and Dirac quantum fields see this singularity. Subsequently we check the results by applying the maximal acceleration existence in Covariant Loop Quantum Gravity described recently and obtain the resolution of singularity. In the process we study the geodesics in the spacetime.

KEYWORDS: exact solution, singularity, scalar field, quantum gravity

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1 Introduction

Curvature singularities that generally appear in solutions of General Relativity signal the limits of validity of this theory. If they are hidden beneath the horizon they are not influencing external observers and the situation is at least practically less serious (although the problem for the theory itself is not diminished). However, naked singularities represent rather undesirable feature (motivating the Cosmic Censorship Hypothesis) of a solution to Einstein equations especially if the associated matter content seems quite ordinary. One expects that the singularities might be removed once quantum gravity is established in one form or another. In the mean time we are trying to probe the singularity using quantum mechanics to determine whether the quantum matter can experience its presence at all. Then we move a step further and use recent results in Covariant Loop Quantum Gravity to determine whether the singularity might survive this specific form of spacetime quantization.

Recently derived Robinson–Trautman solution minimally coupled to a free massless scalar field [1] (broader overview of standard Robinson–Trautman solutions with many references can be found there) led to a relatively wide range of special cases [2] with some peculiar properties. The solution was obtained by generalizing the original form of Robinson–Trautman metric which normally admits only pure radiation and Maxwell field stress energy tensor aligned with the principal null direction or a cosmological constant. The reason was that the scalar field wave equation is incompatible with scalar field whose gradient is aligned. The scalar field had to be considered non-aligned and the Robinson–Trautman metric had to be modified to admit a broader class of energy-momentum tensors.

This study concentrated on the general branch of the solution where all the free constants are nonzero.

Interesting special cases corresponding to vanishing of some parameters were analyzed in [2] and the specific subcases which are all spherically symmetric were put in context of the previously known solutions. In most of these cases the scalar field is still present. One of them is the static spherically symmetric solution with static scalar field which represents a parametric limit of Janis–Newman–Winicour scalar field spacetime [3, 4]. This spacetime is asymptotically flat and scalar field is vanishing at infinity. As observed in Chase theorem [5] (see [6] for recent generalization to large class of potentials) the static configurations with scalar field do not possess regular horizon which is the case here as well. This special solution contains naked singularity which is moreover timelike. It is also sourced by quite ordinary scalar field which seems to be physically realistic source.

Our first approach to the problem of naked singularity presence will be based on the pioneering work of Wald[7], which was further developed by Horowitz and Marolf (HM) [8] to probe the classical singularities with quantum test particles obeying the Klein-Gordon equation in static spacetimes having timelike singularities. Later it was applied in many specific geometries containing singularity [9–18]. The key idea is to find the singular character based on the number of self-adjoint extensions of the evolution operator to the entire Hilbert space including the singularity position. If the extension is unique, it is said that the space is quantum mechanically regular. This is connected to the fact that one can in general select the self-adjoint extension by demanding specific boundary conditions however this cannot be applied in the singularity where we do not have any control over physics and therefore the extension should be unique automatically.

The second approach utilizes recent developments in Covariant Loop Quantum Gravity [19]. It represents singularity investigation on the level of full quantum gravity picture (however this is just one of the candidate theories) or in other words on the level of space-time quantization itself and so it should be considered as more fundamental. The method is based on the appearance of maximal acceleration in this theory [20] coming from quantization in an analogous way to the derivation of minimal area in original Loop Quantum Gravity [21].

2 Self-adjoint extension method

First we present a method for probing singularities with quantum mechanics used in [8]. Consider a static spacetime $(\mathcal{M}, g_{\mu\nu})$ with a timelike Killing vector field ξ^μ [8]. Let t denote the affine parameter along the Killing field and Σ denote a static spatial slice (with singular points removed).

The Klein-Gordon equation can then be written in this form

$$\frac{\partial^2 \psi}{\partial t^2} = \sqrt{f} D^i \left(\sqrt{f} D_i \psi \right) - f M^2 \psi = -A \psi, \quad (2.1)$$

in which $f = \xi^\mu \xi_\mu$ (using the signature $(+, -, -, -)$ of a spacetime metric) and D_i is the spatial covariant derivative on Σ induced from the full spacetime covariant derivative. The

Hilbert space \mathcal{H} , $(L^2(\Sigma))$ is the space of square integrable functions on Σ . Operator A is evidently real, positive and symmetric and therefore its self-adjoint extensions (covering the extension of Hilbert space to encompass the singular point) always exist. If this extension is unique then A is called essentially self-adjoint.

Consider the following equation

$$A\psi \pm i\psi = 0, \quad (2.2)$$

the operator A will be essentially self-adjoint if one of the two solutions of this equation (for each sign of the imaginary term) fails to be square integrable near the singularity. If A is essentially self-adjoint for $M = 0$, it is essentially self-adjoint for all $M > 0$ as well [22]. For simplicity we consider only massless case in Klein-Gordon and Dirac equations.

3 The spacetime with a scalar field

One of the sub cases in [2] is a simple static spacetime with scalar field. The metric corresponding to this solution is

$$\begin{aligned} ds^2 &= dt^2 - dr^2 - (r^2 - \chi_0^2) d\Omega^2, \\ d\Omega^2 &= d\theta^2 + \sin^2\theta d\varphi^2 \end{aligned} \quad (3.1)$$

for which all the Weyl scalars are zero except for Ψ_2 which becomes

$$\Psi_2 = \frac{\chi_0^2}{3(r^2 - \chi_0^2)^2}$$

signaling that it is a solution of algebraic type D . The geometry is obviously static and spherically symmetric. The scalar field is given by

$$\Phi(r) = \frac{1}{\sqrt{2}} \ln \left\{ \frac{r - \chi_0}{r + \chi_0} \right\}. \quad (3.2)$$

The Ricci scalar and the Kretschmann invariant are

$$RicciSc = -\frac{2\chi_0^2}{(r^2 - \chi_0^2)^2} \quad (3.3)$$

$$Kretschmann = 3(RicciSc)^2$$

One can easily see that the singularity at $r = \chi_0$ is naked in this spacetime, either directly from metric or by looking for marginally trapped surfaces. The singularity is pointlike and timelike. When $r \rightarrow \infty$ the scalar field vanishes and the metric (3.1) is asymptotically flat. The area of spherical surfaces $r = const., t = const.$ grows quadratically with coordinate r only far from the central region while close to the singularity $r = \chi_0$ it grows only linearly. It is possible to shift the location of singularity to zero by a coordinate transformation

$$\rho^2 = r^2 - \chi_0^2, \quad (3.4)$$

which results in the metric

$$ds^2 = dt^2 - \frac{\rho^2}{\rho^2 + \chi_0^2} d\rho^2 - \rho^2 d\Omega^2. \quad (3.5)$$

For the subsequent calculation we retain the original form (3.1).

4 Quantum Fields

Now we will use the above method in the specific case of massless scalar field and Dirac field.

4.1 Klein–Gordon Field

The Klein–Gordon equation for a the massless scalar particle is given by,

$$\square\tilde{\psi} = g^{-1/2}\partial_\mu \left[g^{1/2}g^{\mu\nu}\partial_\nu \right] \tilde{\psi} = 0. \quad (4.1)$$

For the metric (3.1) the Klein-Gordon equation becomes

$$\frac{\partial^2\tilde{\psi}}{\partial t^2} = \left\{ \frac{\partial^2}{\partial r^2} + \frac{2r}{r^2 - \chi_0^2} \frac{\partial}{\partial r} + \frac{1}{r^2 - \chi_0^2} \left(\frac{\partial^2}{\partial \theta^2} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} + \frac{\cos \theta}{\sin \theta} \frac{\partial}{\partial \theta} \right) \right\} \tilde{\psi}, \quad (4.2)$$

In analogy with the equation (2.1), the spatial operator A is

$$A = - \left\{ \frac{\partial^2}{\partial r^2} + \frac{2r}{r^2 - \chi_0^2} \frac{\partial}{\partial r} + \frac{1}{r^2 - \chi_0^2} \left(\frac{\partial^2}{\partial \theta^2} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} + \frac{\cos \theta}{\sin \theta} \frac{\partial}{\partial \theta} \right) \right\}, \quad (4.3)$$

Using separation of variables, $\tilde{\psi} = e^{i\omega t} R(r) Y_l^m(\theta, \varphi)$, we obtain the radial portion of equation (4.2)

$$\frac{d^2 R(r)}{dr^2} + \frac{2r}{r^2 - \chi_0^2} \frac{dR(r)}{dr} - \left(\frac{l(l+1)}{r^2 - \chi_0^2} \right) R(r) = -\omega^2 R(r) \quad (4.4)$$

and the equation to be solved is (2.2). So we have an ODE

$$\frac{d^2 \psi_\pm(r)}{dr^2} + \frac{2r}{r^2 - \chi_0^2} \frac{d\psi_\pm(r)}{dr} - \left(\frac{l(l+1)}{r^2 - \chi_0^2} \pm i \right) \psi_\pm(r) = 0. \quad (4.5)$$

This is a Heun (singly) Confluent equation which is obtained from the general Heun equation containing four regular singularities through a confluence process, that is, a process where two singularities coalesce, performed by redefining parameters and taking limits, resulting in a single (typically irregular) singularity [23]. In our case (4.5) we have two regular singularities at $r = \pm\chi_0$ and an irregular one at infinity. The solution for the above equation is expressed using Heun Confluent functions

$$\psi_\pm(r) = C_1 HeunC \left(0, -\frac{1}{2}, 0, \mp \frac{i\chi_0^2}{4}, \eta, \frac{r^2}{\chi_0^2} \right) + C_2 r HeunC \left(0, \frac{1}{2}, 0, \mp \frac{i\chi_0^2}{4}, \eta, \frac{r^2}{\chi_0^2} \right), \quad (4.6)$$

where

$$\eta = \frac{1}{4} \left(\pm i \chi_0^2 - l(l+1) + 1 \right).$$

If we do not consider the subdominant $\pm i$ term in (4.5), the Confluent Heun functions would simplify into

$$\psi(r) = C_1 P \left(l, \frac{r}{\chi_0} \right) + C_2 Q \left(l, \frac{r}{\chi_0} \right), \quad (4.7)$$

where P, Q are Legendre functions of the first and second kind. For analyzing the square-integrability it is worth to know the asymptotic behaviors of the above functions around the singular point $r = \chi_0$. The Legendre function $P\left(l, \frac{r}{\chi_0}\right)$ at $r = \chi_0$ is regular

$$P_l(1) = 1$$

and the Legendre function of the second kind, $Q\left(l, \frac{r}{\chi_0}\right)$, can be written as

$$Q_l\left(\frac{r}{\chi_0}\right) = \frac{1}{2}P_l\left(\frac{r}{\chi_0}\right) \ln \left[\frac{r + \chi_0}{r - \chi_0} \right] - \frac{2l-1}{l}P_{l-1}\left(\frac{r}{\chi_0}\right) - \dots \quad (4.8)$$

The square integrability of the solution (4.7) is checked by calculating the squared norm in the proper function space on each $t = \text{const}$ hypersurface Σ which we consider as $\mathcal{H} = L^2(\Sigma, \mu)$, where μ is measure given by the spatial metric volume element. It is straightforward to show that both solutions are square integrable at $r = \chi_0$ since the logarithmic divergence in (4.8) is compensated by the volume form $([r^2 - \chi_0^2]dr)$ to give a finite limit at $r = \chi_0$ for the integrand. One might be worried that by removing the complex term from the equation we have changed its nature too much. However, as shown in [23] one of the solutions of Confluent Heun equation has (for the specific values of our parameters) logarithmic divergence — as in the case of Q_l — and the other is regular.

4.2 Dirac Field

The Newman-Penrose formalism [24] will be used here to analyze the properties of operator governing the massless Dirac fields (fermions). The Chandrasekhar-Dirac (CD) [25] equations in Newman-Penrose formalism are suitable for this task and are given by

$$\begin{aligned} (D + \epsilon - \rho) F_1 + (\bar{\delta} + \pi - \alpha) F_2 &= 0, \\ (\nabla + \mu - \gamma) F_2 + (\delta + \beta - \tau) F_1 &= 0, \\ (D + \bar{\epsilon} - \bar{\rho}) G_2 - (\delta + \bar{\pi} - \bar{\alpha}) G_1 &= 0, \\ (\nabla + \bar{\mu} - \bar{\gamma}) G_1 - (\bar{\delta} + \bar{\beta} - \bar{\tau}) G_2 &= 0, \end{aligned} \quad (4.9)$$

where F_1, F_2, G_1 and G_2 are the components of the Dirac field wave function, $\epsilon, \rho, \pi, \alpha, \mu, \gamma, \beta$ and τ are the spin coefficients and the "bar" denotes complex conjugation. The null tetrad vectors for the metric (3.1) are defined by

$$\begin{aligned} l^a &= (1, 1, 0, 0), \\ n^a &= \left(\frac{1}{2}, -\frac{1}{2}, 0, 0 \right), \\ m^a &= \frac{1}{\sqrt{2(r^2 - \chi_0^2)}} \left(0, 0, 1, \frac{i}{\sin \theta} \right). \end{aligned} \quad (4.10)$$

The directional derivatives in the Dirac equation are defined by $D = l^a \partial_a$, $\nabla = n^a \partial_a$ and $\delta = m^a \partial_a$. We define auxiliary differential operators

$$\begin{aligned} \mathbf{D}_0 &= D, \\ \mathbf{D}_0^\dagger &= -2\nabla, \\ \mathbf{L}_0^\dagger &= \sqrt{2(r^2 - \chi_0^2)} \delta \quad \text{and} \quad \mathbf{L}_1^\dagger = \mathbf{L}_0^\dagger + \frac{\cot \theta}{2}, \\ \mathbf{L}_0 &= \sqrt{2(r^2 - \chi_0^2)} \bar{\delta} \quad \text{and} \quad \mathbf{L}_1 = \mathbf{L}_0 + \frac{\cot \theta}{2}. \end{aligned} \tag{4.11}$$

Evidently, the spatial parts of \mathbf{D}_0 and \mathbf{D}_0^\dagger are purely radial operators, while $\mathbf{L}_{0,1}$ and $\mathbf{L}_{0,1}^\dagger$ are purely angular operators.

The nonzero spin coefficients for (3.1) are

$$\rho = 2\mu = -\frac{r}{r^2 - \chi_0^2}, \quad \beta = -\alpha = \frac{1}{2\sqrt{2}} \frac{\cot \theta}{\sqrt{r^2 - \chi_0^2}}. \tag{4.12}$$

Substituting these nonzero spin coefficients and using the definitions of the operators (4.11) given above into the CD equations (4.9) leads to

$$\begin{aligned} \left(\mathbf{D}_0 + \frac{r}{r^2 - \chi_0^2} \right) F_1 + \frac{1}{\sqrt{2(r^2 - \chi_0^2)}} \mathbf{L}_1 F_2 &= 0, \\ -\frac{1}{2} \left(\mathbf{D}_0^\dagger + \frac{r}{r^2 - \chi_0^2} \right) F_2 + \frac{1}{\sqrt{2(r^2 - \chi_0^2)}} \mathbf{L}_1^\dagger F_1 &= 0, \\ \left(\mathbf{D}_0 + \frac{r}{r^2 - \chi_0^2} \right) G_2 - \frac{1}{\sqrt{2(r^2 - \chi_0^2)}} \mathbf{L}_1^\dagger G_1 &= 0, \\ \frac{1}{2} \left(\mathbf{D}_0^\dagger + \frac{r}{r^2 - \chi_0^2} \right) G_1 + \frac{1}{\sqrt{2(r^2 - \chi_0^2)}} \mathbf{L}_1 G_2 &= 0. \end{aligned} \tag{4.13}$$

For solving these CD equations, we assume separable solution in the form

$$\begin{aligned} F_1 &= f_1(r) Y_1(\theta) e^{i(kt+m\varphi)}, \\ F_2 &= f_2(r) Y_2(\theta) e^{i(kt+m\varphi)}, \\ G_1 &= g_1(r) Y_3(\theta) e^{i(kt+m\varphi)}, \\ G_2 &= g_2(r) Y_4(\theta) e^{i(kt+m\varphi)}. \end{aligned} \tag{4.14}$$

Here $\{f_1, f_2, g_1, g_2\}$ and $\{Y_1, Y_2, Y_3, Y_4\}$ are functions of r and θ respectively. Additionally, m is the azimuthal quantum number and k is the frequency of the Dirac fields, both are assumed to be positive and real. By substituting (4.14) into (4.13) and using these assumptions

$$f_1(r) = g_2(r) \quad \text{and} \quad f_2(r) = g_1(r), \tag{4.15}$$

$$Y_1(\theta) = Y_3(\theta) \quad \text{and} \quad Y_2(\theta) = Y_4(\theta), \tag{4.16}$$

we reduce (4.13) to only two equations. The important radial part of the Dirac equation becomes

$$\begin{aligned} \left(\mathbf{D}_0 + \frac{r}{r^2 - \chi_0^2} \right) f_1(r) &= \frac{\lambda}{\sqrt{2(r^2 - \chi_0^2)}} f_2(r), \\ \frac{1}{2} \left(\mathbf{D}_0^\dagger + \frac{r}{r^2 - \chi_0^2} \right) f_2(r) &= \frac{\lambda}{\sqrt{2(r^2 - \chi_0^2)}} f_1(r), \end{aligned} \quad (4.17)$$

where λ is the separation constant. For further simplification we introduce a new functions

$$\begin{aligned} f_1(r) &= \frac{\zeta_1(r)}{\sqrt{r^2 - \chi_0^2}}, \\ f_2(r) &= \frac{\sqrt{2}\zeta_2(r)}{\sqrt{r^2 - \chi_0^2}}, \end{aligned}$$

and the equations (4.17) transform into

$$\begin{aligned} \mathbf{D}_0 \zeta_1(r) &= \frac{\lambda}{\sqrt{r^2 - \chi_0^2}} \zeta_2(r), \\ \mathbf{D}_0^\dagger \zeta_2(r) &= \frac{\lambda}{\sqrt{r^2 - \chi_0^2}} \zeta_1(r). \end{aligned} \quad (4.18)$$

or explicitly

$$\begin{aligned} \left(\frac{d}{dr} + ik \right) \zeta_1(r) &= \frac{\lambda}{\sqrt{r^2 - \chi_0^2}} \zeta_2(r), \\ \left(\frac{d}{dr} - ik \right) \zeta_2(r) &= \frac{\lambda}{\sqrt{r^2 - \chi_0^2}} \zeta_1(r). \end{aligned} \quad (4.19)$$

In order to write the above equation in a more compact form we combine the solutions in the following way,

$$\begin{aligned} \Xi_+ &= \zeta_1 + \zeta_2, \\ \Xi_- &= \zeta_2 - \zeta_1. \end{aligned}$$

and square the operators to end up with a pair of one-dimensional Schrödinger-like wave equations with effective potentials,

$$\left(\frac{d^2}{dr^2} + k^2 \right) \Xi_\pm = V_\pm \Xi_\pm, \quad (4.20)$$

$$V_\pm = \frac{\lambda^2}{r^2 - \chi_0^2} \mp \frac{r\lambda}{(r^2 - \chi_0^2)^{\frac{3}{2}}}. \quad (4.21)$$

In analogy with the equation (2.1), the spatial operator A for the massless case is

$$A = -\frac{d^2}{dr^2} + V_{\pm},$$

so from the self-adjoint extension method (2.2) we have

$$\left(-\frac{d^2}{dr^2} + \left[\frac{\lambda^2}{r^2 - \chi_0^2} \mp \frac{r\lambda}{(r^2 - \chi_0^2)^{\frac{3}{2}}} \right] \pm i \right) \psi_{\pm} = 0. \quad (4.22)$$

For finding the exact solutions of the above equation, we ignore the subdominant $\pm i$ part and obtain

$$\psi_{\pm} = C_1 \left(\pm 2\lambda \sqrt{r^2 - \chi_0^2} + r \right) \left(\sqrt{r^2 - \chi_0^2} + r \right)^{\mp \lambda} + C_2 \left(\sqrt{r^2 - \chi_0^2} + r \right)^{\pm \lambda} \quad (4.23)$$

in which λ should be an integer. Obviously, when $r \rightarrow \chi_0$ (which is the singular point) the above two solutions would be finite and their Hilbert space norms near the singular point as well.

5 Geodesic equations

Before going in the direction of Quantum Gravity investigation we need to understand the nature of the singularity a bit more. In this section we want to study the trajectory for a test particle moving on a timelike geodesic. The simplest approach is to use the variational principle or Euler–Lagrange equations for timelike geodesics as follows

$$2\mathcal{L} = g_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu} = \dot{t}^2 - \dot{r}^2 - \left(r^2 - \chi_0^2 \right) \left(\dot{\theta}^2 + \sin^2 \theta \dot{\varphi}^2 \right), \quad (5.1)$$

letting a dot denote derivative with respect to proper time τ . The Euler–Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial x^{\mu}} - \frac{d}{d\tau} \left(\frac{\partial \mathcal{L}}{\partial \dot{x}^{\mu}} \right) = 0 \quad (5.2)$$

give us two conserved quantities, namely the energy (E) and the angular momentum (L)

$$\begin{aligned} \frac{d}{d\tau} \left(\frac{\partial \mathcal{L}}{\partial \dot{t}} \right) &= 0 \Rightarrow \dot{t} = E, \\ \frac{d}{d\tau} \left(\frac{\partial \mathcal{L}}{\partial \dot{\varphi}} \right) &= 0 \Rightarrow \dot{\varphi} = \frac{L}{(r^2 - \chi_0^2) \sin^2 \theta}. \end{aligned} \quad (5.3)$$

We consider motion in the equatorial plane $\theta = \frac{\pi}{2}$. Substituting (5.3) in (5.1), we obtain

$$E^2 - \dot{r}^2 - \frac{L^2}{(r^2 - \chi_0^2)} = 1 \quad (5.4)$$

and for a qualitative analysis of geodesics we employ the standard effective potential method. We can write the equation for radial velocity in the following form

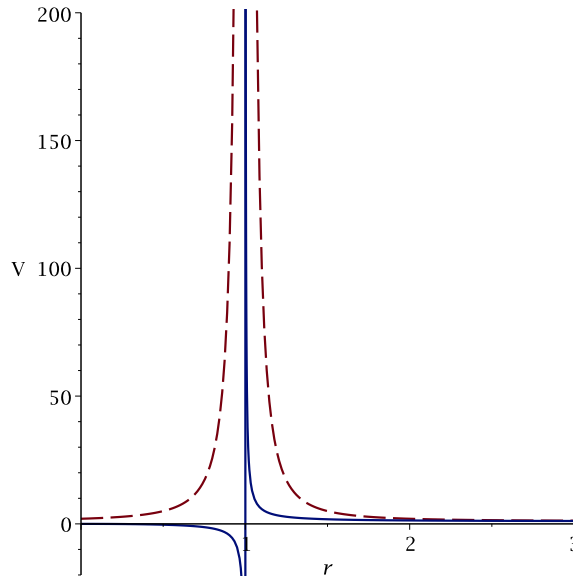


Figure 1. Plot of effective potential (5.5) for $L = 1$ and $\chi_0 = 1$ (solid line) compared with standard centrifugal barrier (dashed line) with respect to center shifted to position $r = \chi_0$ instead of $r = 0$.

$$\dot{r}^2 + V_{eff} = E^2 ,$$

$$V_{eff} = 1 + \frac{L^2}{(r^2 - \chi_0^2)} . \quad (5.5)$$

The effective potential is plotted in Figure 1. It is evidently repulsive and acting similarly to centrifugal barrier (in flat space) and in fact its origin is similar. Compared to centrifugal barrier it allows particle to travel closer to the origin (at $r = \chi_0$). Note that for vanishing angular momentum l the radial velocity is constant so the radial particles (or observers) are traveling like in a flat space with constant velocity.

6 Covariant Loop Quantum Gravity

So far we have probed the singularity just by quantum test fields so the results might not be convincing or even correct from the nonperturbative point of view. To proceed further we should consider the quantum gravity picture. When the static spacetime possesses horizon covering the central spatial singularity one can use the Loop Quantum Cosmology method since the spacetime below horizon (which is no longer static) can often be mapped onto some symmetric cosmological model whose singularities are generally resolved. Our spacetime is naked and the singularity is timelike so we cannot use this trick. Instead we can apply the recent discovery on the level of Covariant Loop Quantum Gravity [20] that quite generally the singularities are resolved due to the maximal value the acceleration of observers can attain. The derivation is based on considering Rindler observers but is later

applied to cosmology with the characteristic acceleration being the mutual acceleration of nearby comoving observers. Here, we consider essentially the same quantity, the relative acceleration with respect to radial geodesic as given by the geodesic deviation equation. This as well measures the tidal forces acting on an object approaching the singularity.

The fourvelocity of a radial geodesic (considered in the equatorial plane for simplicity) is described by

$$u^\mu \partial_\mu = \sqrt{(u^r)^2 + 1} \partial_t + u^r \partial_r \quad (6.1)$$

with radial velocity u^r being a constant. Deviation vector will be assumed in the form $\delta = \delta^\phi \partial_\phi$. The geodesic deviation equation then assumes the following form

$$\frac{D^2 \delta^\alpha}{d\tau^2} = -R^\alpha{}_{\beta\gamma\sigma} u^\beta \delta^\gamma u^\sigma = -\frac{\chi_0^2 (u^r)^2}{(r^2 - \chi_0^2)^2} \delta^\alpha. \quad (6.2)$$

Evidently, the tidal force grows unbounded when approaching the singularity even though the radial geodesic observer is not accelerated with respect to asymptotic observer. As a measure of acceleration we will use the invariant norm of (6.2) with respect to unit of separation

$$a = \frac{\chi_0^2 (u^r)^2}{(r^2 - \chi_0^2)^{3/2}}. \quad (6.3)$$

According to [20] the acceleration is bounded by a maximum value $a_{max} \sim \sqrt{\frac{1}{8\pi G\hbar}}$ (in nongeometric units). This result is moreover derived in covariant theory unlike previous upper bounds in acceleration [26]. Inspecting (6.3) one immediately sees that the upper bound on acceleration means that the divergent factor $(r^2 - \chi_0^2)^{-1}$ appearing in curvature scalars (3.3) is bounded and therefore the singularity is resolved on the level of Covariant Loop Quantum Gravity. Accordingly the tidal forces are bounded and an object can in principle survive the fall into the singularity. However, the bound is extremely large so it is hard to imagine any realistic object that would not be crushed.

So the infinities connected with the presence of singularity are cured in this scenario.

7 Conclusion and final remarks

We have shown that for both Klein–Gordon field and Dirac field the solutions of (2.2) are square integrable which means that the corresponding operators in both cases are not essentially self-adjoint and therefore the problem is quantum mechanically singular. So the probe fields still see the singularity in this case.

In the case of Covariant Loop Quantum Gravity the maximal acceleration existence provides the means to effectively remove the singularity. Since this method is based on quantum description of spacetime one should give it preference over the first approach where the spacetime itself is classical only the probes are quantum. On the other hand we have not performed complete quantization of the spacetime and one should still view this result as an indication of singularity resolution in this theory rather than a complete proof.

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